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FIXED-TO-MOBILE SUBSTITUTION IN THE US, EU, AND CHINA

Forecasting technology diffusion using the Lotka-Volterra Competition model

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Bachcelor’s Thesis
Instructor: Roman Stepanov
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Declaration

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ABSTRACT OF
BACHELOR'S THESIS

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Objectives

The first purpose of this thesis is to test the performance of the Lotka – Volterra Competition model in forecasting demand for technologies. Secondly, the paper aims to determine the interrelationship between the markets and their expected behaviors based on population theories. Thirdly, it attempts to gauge the similarities and differences of market behaviors in the most developed economies based on GDPpc as of October 2018.

Summary

Total annual subscription for each market was used to perform in-sample forecasts. Parameterization was obtained using the Gauss-Newton non-linear least squares method with the Marquardt algorithm. Then, the stable equilibria were shown in the interactive outcome graphs, which indicate that the theoretical suggestions are well-supported by historical market patterns.

Conclusions

The results indicate high fitting performance (R-squares>0.98) with estimated data close to that of actual observations. Despite data complications, the model has a good degree of accuracy. The competitive relationships for the US, the EU, and China are suggested to be amensalism, amensalism, and pure competition, respectively. Over time, mobile phones will substitute fixed – line phones and obtain maximum growth.

Key words: *forecasts, non-linear models, diffusion, equilibrium analysis, mobile phones, mobile technology, telephones*

Language: English

Grade:

A. Introduction

Within the last two decades, the world has witnessed a great influx of innovations that redefined the technological scene, including the Walkman, the iPod, the laptop, and so on. Following this replacement of incumbent technology is a large phenomenon known as wireless broadband substitution, which explains the shift in smart products to be cordless cable-free. This area of wireless broadband includes two major influences that have been of public and academic interest: cord – cutting and fixed – to – mobile substitution.

To provide an illustration of the fast-moving shift from fixed to mobile technology, within a 10 year period, from 2006 to 2016, Spain's mobile broadband sector's revenue increased by almost 10 times, while that of fixed broadband has remained stable (Comisión Nacional de los Mercados y la Competencia, 2017). According to the International Telecommunications Union, since 1993, worldwide subscription to mobile broadband has rapidly soared from 34 million to 8160 million (ITU, 2018). Similarly, demand for wireless communication, specifically mobile phones, have been rising significantly. In Asia Pacific alone, mobile cellular subscription has gone from 833 million in 2005 to 4351 million in 2017 (ITU, 2018).

Gradually, the spread of mobile phones has become recognized for its importance as and indicator for economic growth. The degree of stagnation in economic development could be attributed to unsatisfied demand for telecommunication services (Gruber, 2001). Thus, the study of fixed – to – mobile substitution has gained traction in the academic community. Many papers have been devoted to understanding, determining, and predicting the spread of this phenomenon. While case – by – case analyses are abundant (See for example: Kumar, et al., 2007; Kalogiratou, et al., 2013; Wulf, et al., 2013), cross-regional studies are few in number.

This thesis aims to contribute to existing knowledge on fixed – to – mobile substitution and its gap in cross-regional studies by examining the competitive effects between the mobile phone market and the telephone market in three developed economies – the US, the EU, and China. To attain this goal, the Lotka – Volterra Competition model, originally from population biology, will be employed to produce in-sample forecasts. Then, based on the results from parameterization, the competitive relationship between these markets will be determined. An equilibrium analysis will be performed for each region to gauge whether the theoretical suggestions of the model are in accordance with true data.

B. Literature Review:

1. Background

As a result of technological advancements, broadband, mobile devices, and online networks have become more prevalent and, thus, considerably boosted the amount of time households spend on the internet (Brandley & Bartlett, 2011). Consequently, companies have gradually ascribed a higher value to broadband network, leading to the expansion of cellular services, and the convergence of information, analytics, entertainment, and commerce (OECD, 2016; Cunningham, et al., 2016). An average consumer now has access to a greater range of content on multiple devices that are easily accessible and available.

The above changes combined with the increasing rate of media digitization have led to the emergence of ‘cord-cutting’. During the past five years, television newcomers such as Netflix, Hulu, and Amazon Prime have challenged traditional cable companies, referred to as Multiple System Operators. While these companies offer similar services via wireless connection, many of them have surpassed traditional TV in terms of price, content variation, and connectivity. Some firms who have noticed these potentials, including premium, basic cable, broadcast networks, began to create streaming services. Such services are commonly known as over-the-top (OTT) media (Cunningham, et al., 2016; Fuduric, et al., 2018; Tefertiller, 2018). Thus, by alternating a traditional cable for a broadband connection, viewers are said to be practicing video cord-cutting (Katz, & Le Champion, 2013).

Before the arrival of cord-cutting, global markets and academics have detected a rather similar trend referred to as ‘wireless substitution’ or ‘fixed-mobile substitution’ (FMS) that also occurred in the telecommunications industry. Telecommunication analysts referred to the term to depict the replacement of fixed-line phones, which operated via a hard-wired system, for broadband-connected devices, such as the mobile phone (Albon, 2006; Shin, 2012). During the 1980s when cellphones were first introduced, the public did not view them as a worthwhile alternative to landlines due to their bulky size, low transmission quality, and high price level. The spread of mobile services only accelerated when telecommunications authorities were concerned about the lag in competition in local areas (Rodini, et al., 2003). After the Global System for Mobile communications (GSM) was deployed by Finland in 1991, mobile devices became widely manufactured with companies such as Nokia, Motorola, Sony and so forth. As a common economic result, prices dropped and penetration rates soared (Harald, 2005). Gradually, as cellular service providers and consumers increased, scholars began to notice the substitutability of mobile phones to traditional landlines.

In 2012, Barth and Heimeshoft observed the development of FMS with evidence from 16 European countries and concluded that the fixed and mobile markets had been converging and becoming closer substitutes (Heimeshoff & Barth, 2012). This is consistent with previously released data not just in the EU alone but also in numerous other areas. From 2000 to 2017, while global figures for fixed-line phone usage saw a steep decline, the market penetration rate for mobile phones skyrocketed. For instance, in the UK alone, the total minutes of fixed voice calls were almost three times fewer in 2017 compared to 10 years ago (Ofcom, 2018). Similar patterns were also noted in developing countries. During the same period, China's mobile subscriptions experienced an average of a 15% growth while per year, while that of fixed-line operators gradually reduced (MIIT, 2018). Even in a region that had late access to technological improvements such as Africa, in 2008, the number of mobile cellular subscriptions reached 246 million, approximately 23 times greater than that of telephone lines (Aker & Mbiti, 2010).

These fast-moving figures convinced researchers that the prevalence of FMS could render landlines obsolete. Gradually, a number of articles investigating the interrelationship of mobile and fixed broadband began to surface. After a decade since its development in the early 2000s, most studies have come to agree that mobile phones would replace traditional telephones' position to dominate the telecommunications industry (see for example: Kim, et al., 2006; Wulf, et al., 2013; Grzybowski, 2014). Recently, it has been reported that wireless substitution has entered a second phase in which the number of mobile offline calls and texts face a reduction due to social networks (Shin, 2012)

As the early occurrence of fixed – to – mobile substitution has granted it more academic interest, there is a wide range of methodologies for studying this phenomenon. From correlational tests based on consumer surveys to econometric models, many authors have attempted to dissect and predict market behaviors from different angles. One particular branch of studies that stand out for its reliability, fitting performance, and ease of operation is forecasting based on theories of population biology and technology diffusion. The following section will look into the foundations and main pillars of these concepts and explain why they could be effectively utilized in understanding economic matters.

2. Population dynamics

a. Definition

Population biology, an interdisciplinary field inspired by ecology and evolutionary biology, originated from a wide range of areas. For instance, "…in taxonomy, in studies of the geographical distribution of organisms, in natural history studies of the habits and interactions between organisms and their environment, in studies of the characteristics of organisms are inherited from one generation to the next and in theories which consider how different types of organisms are related by descent" (Niel, 2004 : 1). Consequently, the academic community have generally used the term 'population dynamics' to convey the changes, whether short- or long-term, of populations. These changes could include those of weight, age distribution, sex ratio, behavior and so forth. Population dynamics also aims to explore the process by which these changes occur as well as their biological and environmental determinants (Wilson & Ferris, 1987; Michalakelis, et al., 2011).

b. Founding principles

Berryman (1997) classified four fundamental principles of population dynamics. The first principle, also referred to as the 'Malthusian Law', was developed by Malthus in 1798. The law stated that populations are able to grow in a geometrical ratio when unchecked (Malthus, 1798). It was later realized that there is a limit to the growth of populations as many species do not reproduce continuously and, so, 50 years later, Verhulst adjusted the principle by creating a logistic growth curve, meaning that the growth of a population is now regulated by its density, and that there is an inverse relationship between them due to competition for the amount of resources. Verhulst law is referred to as the 'density dependence' or 'density-induced negative feedback', and is regarded as the second important principle in the literature of population dynamics. In contrast, in the 1930s, Warder Clyde Allee, examined a small group of goldfish and concluded that, when density is low, cooperation could generate more benefits to sustain the population. For certain groups of animals that rely heavily on collective hunting or defense to persist, this assumption holds true. This effect is in turn named the 'Allee effect', and is the third notable principle. The last one, the principle of delayed density dependence, was introduced by Hutchinson in 1948. According to his research, rather than letting population growth be dictated by density, several species would control it with a negative feedback operated with a time lag, creating population cycles – the change in population that is predictable over a period of time.

c. Application – technology diffusion

Numerous applications have arisen from population dynamics and its principles. For instance, the logistic growth curve, which is the result of Verhulst logistic theory of growth, has been used by Carlson (1913) to model the growth of yeast, or by Fisher and Pry (1971) to study market penetration for a new technological product. Aside from intrinsic growth, another corresponding academic interest is the study of individuals interacting with one another in their own species (intraspecific competition), or with those of other species (interspecific competition). Similar to how animals in an environment compete for the same resource to ensure their survival, firms in their respective industries behave the same way to sustain their positions that are often illustrated by indicators such as revenue share, market share, stock price, and so on. In this age of technological liberation, concepts from population dynamics have been borrowed to enhance our understanding of technology diffusion – the development of a product's life cycle and sales (see for example: Kreng & Wang, 2010; Wang & Wang, 2017; Zhang, et al., 2018). For executives, estimations on a product's life cycle provide a solid foundation on which informed decisions, such as those regarding revenue maximization or cost minimization, could be made. For policy makers, by comparing the evolution curves among technologies, an effective allocation of scarce national resources could be generated. Therefore, the models that have been developed to research competing species and technological diffusion are undeniably an important economic tool.

3. Models for competing species and technology diffusion

This section will present a review of the models that serve to forecast growth with respect to their theories and consequently justify for the use of those developed for competing species. Then, it will analyze and compare the most prominent models that have been applied to the study of technology diffusion in competitive industries.

a. Single species

Verhulst (1838) published an equation that embeds his logistic growth theory:

$$\frac{dN}{dt} = rN - aN^2$$

where *N(t)* is the size of the population at time *t*. For example, it could be the number of individuals of a species, or the number of subscribers to a product. *r* is the constant growth rate (intrinsic growth rate), and a is the density dependent crowding effect.

Later, the equation was modified by Raymond Pearl and Lowell Reed with *K* as the carrying capacity, or the maximum population density that can be supported by the environment. The rate of change of a population is proportional to the size of the population. Thus, in the absence of competition, the growth of a single species, similar to that of a single product or technology, could be modeled by the following logistic equation:

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) \quad (1)$$

This equation has led to the development of several models for demand estimation. For instance, the family growth model (Fisher & Pry, 1971) or the Gompertz model (Rai, 1999). For single species, these methods provide a logical application. However, when there is a mutual presence of more than one population competing for similar resources, it is expected that they will affect each other's growth rate and saturation rate. For instance, in a competitive or oligopolistic market, each firm's share will be reduced due to their constant strive for resources such as profit, or due to the entrance of new competitors (Michalakelis, et al., 2011). Equation (1) is thus insufficient as it does not account for the interplay among species and the magnitude of each species' existence on each other's survival.

b. Competing species – the Lotka Volterra model:

The first model that included parameters which measure the intensity of inter-species interference was the Lotka-Volterra competition (LVC) model, which was initially conceptualized by Vito Volterra after observing population changes of sharks and fish in the Adriatic Sea. To understand the intuition this model, it is crucial that the original equations be examined.

$$\frac{dN_1}{dt} = r_1 N_1 \frac{(K_1 - N_1 - \alpha_{12} N_2)}{K_1}$$

$$\frac{dN_2}{dt} = r_2 N_2 \frac{(K_2 - N_2 - \alpha_{21} N_1)}{K_2}$$

In this set, N denotes population size and t is time. So the change in population of one species over time is dN/dt. In the absence of competition, r, describes one species's unlimited growth rate. Thus, the first term *rN*, explains the magnitude of a species's growth when it is not confined by competition. Similar to the logistic growth function, K is the carrying capacity or the maximum population density that could be supported by the environment. Thus, as a population approaches the carrying capacity, the term (K-N) will be smaller. The logistic function thus decribes how the growth rate depends on population density, as when N increases but K stays the same, (K-N) or the numerator decreases and with that, the change in growth will narrow down as N approaches K. This is similar to the well-known basic economic concept of diminishing marginal returns, where due to a fixed input, the pay off from an increase in a variable input will decline over time.

The main difference between the logisitc system and the LVC model is the competition coefficient, or α. α_{12} denotes the interference of species 2 on species 1 and vice versa for α_{21}. Thus, the total product of competition is represented by α multiplied by N of the other species. When α_{12} is less than 1, this means that the effect of species 2 on 1 is less than the effects of the effect of species 1 on its own members. The same is also true for when α_{21} is less than 1.

In 1997, in an effort to make the LVC system suitable for studying technological interactions, Pistorius & Utterback (1996) suggested the following set of two non-linear and first-order equations:

$$\frac{dX}{dt} = a_1X - b_1X^2 + c_1XY \tag{2}$$

$$\frac{dY}{dt} = a_2Y - b_2Y^2 + c_2YX \tag{3}$$

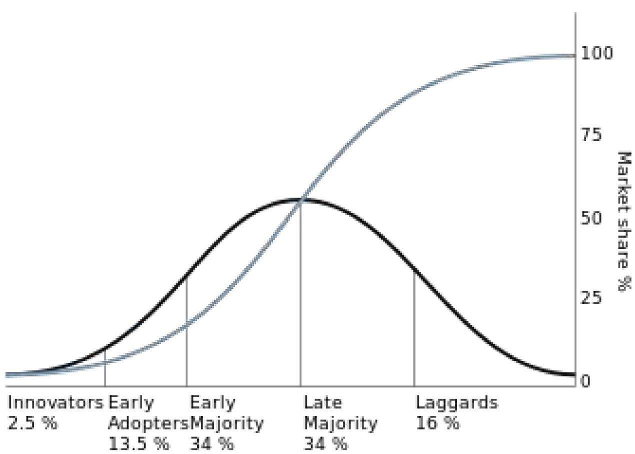
where *X* and *Y* represent the population of two competing species at time *t.* The derivatives *dX/dt* and *dY/dt* are their respective performance change rates. In both equations, there are parameters that explain the growth conditions that have taken into account competitive influences due to coexistence. The term a_i represents unlimited growth rate, which could also be interpreted as the technology performance independent growth rate. The growth of this term could include, for example, R&D investment or policy encouragement (Zhang, et al., 2018). b_i represents the self-crowing effect, or the scale of growth space in the environment. This could arise from resource limitation. The next parameter, c_i explains the positive or negative effect of one species to another and their level of interdependency. Modis (1999) used these terms to measure product attractiveness (a_i), market niche (b_i), and interference in a competitve market (c_i). Modis also provided a method to classify competitve roles, which will be discussed in the methodology part of this thesis, based on the signs of c_i.

c. Technology diffusion – the extended Bass model

The first new product diffusion model was built by Bass in 1969 based on the assumption that the growth of a new technology mimics an S-curve. On one end, the population density of a species exponentially increases due to fertility. The other end exhibits exponential decline until the population reaches a growth rate that is equal to zero (Bass, 1969). This point of stabilization represents the carrying capacity (*K*) of the environment - the added term in Verhulst's law of density dependency. The equation could be specified as follows:

$$dt = \left(p + q\frac{X_t}{m}\right)(m - X_t)$$

where *dt* is the new product purchases at time *t*. *m* denotes market potential, or the upper bound for diffusion. *q* descrbes the influence of adopters and potential adopters on the level of adoption. *p* is the degree to which external influences affect adoption.



The idea behind the Bass model is best illustrated with the S-shaped innovation adoption curve. When a new product or technology is first introduced and there are not many adopters, the diffusion cruve starts off slowly. As more users know about and consume it, the growth curve rises rapidly. Finally, the curve flattens out at the end, showing slow growth once again.

The main inconvenience of the Bass model is that it needs to be applied to a monopolistic market, which is highly unsuitable for forecasting competitive effects. Also, the parameters of this equation do not account for inter-species influences on growth (Wulf, et al., 2013; Wang & Wang, 2016; Tsai, 2017; Zhang, et al., 2018). These are the reasons many researchers have chosen to opt out the use of the model. However, in 2010, it was proposed that the model would include two new terms (Libai, et al., 2009), and, thus, be rewritten as:

$$dt = \left(p + q\frac{X_t}{m_x} + s\frac{y_t}{m_y}\right)(m_x - X_t) \tag{4}$$

where m_i inteprets the potential production for population *I*, and *s* is the level to which a product's adoption is influenced by interferences between potential adopters and adopters.

d. Scope of application

- The Lotka – Volterra Competition model

The LVC model has been applied thoroughly in both economics and finance, and many academics have praised its practicality and effectiveness for forecasting demand and competitive effects in an ologopolistic or comeptitive market (see for example: Kim, et al., 2006; Li & Tsai, 2011; Gupta & Jain, 2016). Modis (1999) used the model to estimate the interrelationship between stocks and bonds as if they were species rivaling for investors' resources. The results showed that there had been a shift from a symbiotic relationship to a 'predetor-prey' one. In 2005, the same experiment was carried out to study the Korean Stock Exchange (KSE) and Korean Securities Dealers Automated Quotation (KOSDAQ). The authors suggested that these stock markets progressed from a symbiotic to a pure competition relationship (Lee, et al., 2005). With respect to products and technologies, Wang and Wang (2017) applied the model to smartphone operating systems, namely Android and iOs, in Taiwan, and concluded that they shared a predetor-prey relationship with iOs being the predator and Android being the prey. During the same period, another study also indicated that the Taiwanese and Chinese motherboard industries experience a similar relationship, with the former being the predator to the later (Tsai, 2017).

With regards to the topic of this thesis, the model has yet to be applied to analyze potential competitive effects of cord-cutting, but it has substantially aided the study of fixed-to-mobile substitution. The first application of the LVC model on the mobile phone market was conducted in Korea, where it was show that the PCS market and cellular market shared a type of commensalism in which PCS benefited from the other's existence (Kim, et al., 2006). In Czech and Slovak Republics, however, the opposite was captured. From 1948 to 2009, fixed lines acted as a predator to their prey – voice service subscribers (Bakaz & Willians, 2012). In Guatemale, it was suggested that the relationship between mobile and fixed-line services has shifted from pure competition to amensalism (Avila, et al., 2018). Depending on the region, each applicaction of the Lotka Volterra model has yielded different results, even if it might seems that total subscriptions for one type of service is greater than the other.

- The Extended Bass model

The Bass model is famous for being a powerful tool for demand forecasting and especially for technological diffusion, as it was originally intended to serve this sole purpose. Previous literature has recorded its applications with numerous products and tecnologies. To provide an illustration, in 2010, it was used to forecast the global adoption of crystal display televisions. The results demonstrated that, for both 37-inch and 32-inch types, the Bass model presented shipment records that were exceptionally close to actual data (Tsai, et al., 2010). Likewise, two other studies, one of silicon wafers adoption and the other of computer shipments in Taiwan, found that the simulated data and the realistic data share a similar trend (Chiang & Wong, 2011; Chiang, 2012).

Regarding mobile telephony, the model has also often been used to forecast demand, factors affecting it, and whether there would be a substitution effect between two products or services. For example, an earlier study applied the Bass model on Taiwan's phone market and demonstrated that mobile telephony was a substitute for fixed-line telephony (Chu, et al., 2009). For BRIC countries, the model produced a good fit and signified that each country had its own profile of diffusion. Interestingly, in India, word-of-mouth was noted as a highly influential factor of mobile communication diffusion as the q value was exceptionally high (Kumar, et al., 2007). More recently, in 2012, the model was applied to predict the diffusion of 2G and 3G mobile services in China. The results implied that Chinese mobile operaters should increasingly deploy 3G services as the market size would increase. In this particular case, the authors applied the model on the country as the whole as well as on specific regions. Although the model performed well, it was noticed that the provincial forecasts were better fitted than the national forecasts. The underlying reason was the the total sum of square errors were greater for data for the whole of China, which is pratical and thus does not reduce the reliability of the analysis (Lim, et al., 2012)

e. Forecasting reliability and accuracy

Unlike the LVC model, the Bass model unfortunately does not provide a comprehensive framework for estimating competitive influences between species. Despite this limitation, it is still possible to utikize the Bass equations to explore the direction in which products are influencing each other. For example, one paper revealed that both models interpreted the relationship between fixed- and mobile- broadband similarly, that the rise in consumer portion for mobile uses negatively impacts the diffusion of fixed broadband (Brenner & Wulf, 2013). Also, the model has been acclaimed for its pioneering role in technological diffusion forecasting. Hence, it is perhaps more suitable to compare these equations in terms of their degree of fitness in predicting diffusion patterns.

Although the LVC model has been well recognized for its performance on competition analyses, it is arguably the optimal method for growth forecasting. It has been suggested in a previous study on wireless substitution that the LVC model had a lower fit (R^2) than the extended Bass model, and that although tests on both models indicated that there was a reverse relationship between fixed and mobile broadband, the results from the former one were less reliable (Brenner & Wulf, 2013). Despite these findings, it has also been found that the Bass model is inferior to the LVC model in a competitive setting. For instace, a research of three companies in Korea inferred that the LV model's curve fit was closer to the true data (Chang, et al., 2014). The same could be said for the Guatemale case in which the author put forward the Bass model and the Gompertz model as functional alternatives to the LVC model. However, after testing their fitting performance via means of R^2, RMSE (mean absolute percentage error), MAPE (root mean squared error), and AIC (Akaike information criterion), the authors concluded that a logistic model, such as LVC, would produce the best diffusion prediction (Avila, et al., 2018). A similar result was noticed in Gupta and Jain’s test for the Indian market of incumbent technologies and new innovations, althought their research only compared the LVC model and the Gompertz model.

In light of these conflicts, the following paper will employ only the LVC competition model due to the following reasons. Firstly, although there is proof that in some cases, the Bass model outperforms the LVC model, the reliability and accuracy indicators for the LVC in most markets were still high. Out of the aforementioned literature, the one that is most critical of the LVC model is that of Brenner and Wulf (2013). Their results showed that the LVC parameters for mobile broadband had an R-squared well below 0.5. Also, the capacity parameter was set to 0, implying that the mobile broadband market’s niche size has no limitation. Resulting from this, the LVC model was regarded as unfitting, and no reliable implication could be drawn with respect to competition. However, it should be noted that Brenner and Wulf used a sequential programming algorithim in SPSS to derive these parameters, unlike most other authors who adopted the Gauss-Newton non-linear least squares method with the Marquardt algorithim in Eviews or SAS (Statistical Analysis System) (See for example: Kim, et al., 2006; Chu, et al., 2009; Kreng & Wang, 2010) . Interestingly, in another 2013 paper, Brenner and Wulf tested the LVC model with the Marquardt algorithim and arrived at reliable parameter estimates with both R-squared greater than 0.95.

Secondly, as has been briefly discussed earlier, the Extended Bass model built on Bass’s 1969 model that assumes an S-shaped growth curve. In reality, many technologies and product do not follow this diffusion pattern, especially when there exists interference from a close substitute in the same market. Most of the studies that apply the LVC model to competitive, non-monopolistic markets mention this issue of the Bass model as well . With respect to fixed-to-mobile substitution specifically, a large number of authors have also highlighted this problem and thus opted for the LVC model. Moreover, the application of the Bass model has mostly been on high technologies and their incumbent counterpart as they exhibit strong, dominant growth rate and are regarded as possible substitution. For an environment in which the substitutability of one species for another is unknow, such as the telephony market that is investigated in this paper, the LVC model is more suited as it offers more interrelationship classification.

Figure 1: theoretical framework

Based on the previous discussion on related literature, this paper will focus on the eligibility of the Competitive Lotka-Volterra model as a method of determining the growth and interrelationship of fixed-line telephony (species 1) and mobile phones (species 2) in the United States, the European Union, and China. Thus, to pursue these purposes, the following research questions are proposed:

1. What are the levels of accuracy and reliability of the LVC model?
2. What is the degree of fitness of the forecasted and actual data?
3. How high is the degree of competition between the selected species?
4. Is there a degree of substitution in any scenario?
5. Would there be a stable equilibrium at which both species could coexist?

The thesis will also advance with the following hypothesis, which will be more clearly defined in the next section:

- H1: the diffusion of mobile phones does influence the growth of telephones in each region.
- H2: mobile phones do not suffer from the existence of telephones.

C. Methodology

To unfold these issues, the thesis will advance as follows:

Step 1: Data, assumptions, and model specification

Step 2: Parameter estimation
using Gauss-Newton non-linear least squares (NLS) method with Marquardt algorithm

Step 3: Results and reliability interpretation
using R-squared, MAE, and MAPE

Step 4: Competitive relationship interpretation

Step 5: Equilibrium analysis

Figure 2: Conceptual Framework

1. Specification

a. The Lotka-Volterra equations

Revisiting the interpretation of Pistorius & Utterback (1996) (Eqs. (2) & (3)), the Lotka-Volterra model can be expressed as follows:

$$\frac{dX}{dt} = a_1X - b_1X^2 + c_1XY = X(a_1 - b_1X - c_1Y) \quad (5)$$

$$\frac{dY}{dt} = a_2Y - b_2Y^2 + c_2YX = Y(a_2 - b_2Y - c_2X) \quad (6)$$

where:

- X, Y: population of two competing species at time *t*.
- a_i: logistic growth parameter for species *I* when it is living alone.
- b_i: limitation parameter of the niche capacity for species *i*.
- c_i: interaction parameter

To utilize discrete time data, equations (5) and (6) need to be transformed into a discrete time version. Leslie (1957) suggested the following interpretation, which has been widely employed in studies of different markets (See for example: Lee, et al., 2005; Chiang & Wong, 2011; Chang, et al., 2014). The difference equations are as follows:

$$X(t+1) = \frac{\alpha_1 X(t)}{1+\beta_1 X(t)+\gamma_1 Y(t)} \tag{7}$$

$$Y(t+1) = \frac{\alpha_2 Y(t)}{1+\beta_2 Y(t)+\gamma_2 X(t)} \tag{8}$$

where α_i and β_i are the logistic growth of the species *i* when it is living alone. γ_i, which is positive, is the interference of one species' growth on that of the other.

To arrive at equations (7) and (8), it is necessary to revisit the logistic differential for the growth of a single species, or equation (1):

$$\frac{dN(t)}{dt} = rN(t)\left(1-\frac{N(t)}{K}\right)$$

where *r* is the difference between the birth rate *(a)* and the death rate *(d)* of a species. It also denotes the intrinsic growth rate if there is no restriction of space or food.

Divide both sides by $N(t)\left(1-\frac{N(t)}{K}\right)$

Yield $\frac{dN(t)}{dt}\frac{1}{\left(1-\frac{N(t)}{K}\right)} = r$ whose integral will be taken. (9)

- Partial fraction expansion:

$$\frac{A}{N(t)} + \frac{B}{1-\frac{N(t)}{K}} = \frac{1}{N(t)\left(1-\frac{N(t)}{K}\right)}$$

$$\Leftrightarrow \frac{A-\frac{A}{K}N+BN(t)}{N(t)\left(1-\frac{N(t)}{K}\right)} = \frac{1}{N(t)\left(1-\frac{N(t)}{K}\right)}$$

$$\Leftrightarrow \frac{A-\frac{A}{K}N+BN(t)}{N(t)\left(1-\frac{N(t)}{K}\right)} = \frac{1+0N(t)}{N(t)\left(1-\frac{N(t)}{K}\right)}$$

$\Leftrightarrow A = 1 \; and \; thus \; -\frac{A}{K} + B = -\frac{1}{K} + B = 0 \Leftrightarrow$ *B = 1/K* (10)

Given (10), equation (9) can be rewritten as:

$$\left(\frac{1}{N(t)} - \frac{-\frac{1}{K}}{1 - \frac{N(t)}{K}}\right)\frac{dN(t)}{dt} = r$$

$$\Leftrightarrow \frac{1}{N(t)}\frac{dN(t)}{dt} - \frac{-\frac{1}{K}}{1-\frac{N(t)}{K}}\frac{dN(t)}{dt} = r$$

$$\Leftrightarrow \ln|N(t)| - \ln\left|1 - \frac{N(t)}{K}\right| + c = rt + c$$

Assume that *0<N(t)<K* so that $|N(t)|$ is always >0 and $\left|1 - \frac{N(t)}{K}\right|$ >0

$$\Leftrightarrow ln\left(\frac{N(t)}{1-\frac{N(t)}{K}}\right) = rt + c$$

$$\Leftrightarrow \frac{N(t)}{1-\frac{N(t)}{K}} = ce^{rt}$$

$$\Leftrightarrow \frac{1-\frac{N(t)}{K}}{N(t)} = ce^{-rt}$$

$$\Leftrightarrow \frac{1}{N(t)} - \frac{1}{K} = ce^{-rt}$$

$$\Leftrightarrow \frac{1}{N(t)} = ce^{-rt} + \frac{1}{K}$$

$$\Leftrightarrow N(t) = \frac{1}{ce^{-rt}+\frac{1}{K}} = \frac{K}{1+ce^{-rt}} \qquad (K = \frac{r}{a}) \tag{11}$$

where *K* is the upper asymptote and *c* is the initial state of the species.

If we let $\lambda = e^{r} = e^{b-d}$ then we could derive from (11):

$N_{t+1} = \frac{\lambda N_t}{1+\alpha N_t}$ where $\alpha = (\lambda - 1)/K$ (12)

Now if there are two species in the same environment, we must introduce the terms for their influence on each other's growth. Equation (12) could thus be rearranged to include γ_i which defines the magnitude of the rate of increase on one on the other. The total effect could, as a result, be represented by $\gamma_i \, N_j(t)$.

$$N_1(t+1) = \frac{\lambda_1 N_1(t)}{1 + \alpha_1 N_1(t) + \gamma_1 N_2(t)}$$

$$N_2(t+1) = \frac{\lambda_2 N_2(t)}{1 + \alpha_2 N_2(t) + \gamma_2 N_1(t)}$$

The set of equations above, given by Leslie, is essentially similar to equations (7) and (8). For ease of coding, this study will use the couple of (7) and (8).

b. Data and assumptions

The chosen data sets for this study were retrieved from The World Bank's world development indicator and the International Telecommunications Union's ICT Development Report. Total annual subscriptions for mobile phones and telephones in the United States, the European Union, and China were used to estimate diffusion were extracted within the periods of 1984-2017, 1980-2017, 1987-2017, respectively. These regions have been chosen as they are currently the top 3 largest economies based on GDPpc (PPP) as of October 2018 (International Monetary Fund, 2018). Although there is more historical data available for telephone usage in all selected regions, these timeframes have been chosen to accommodate the lack of cumulative data for mobile phone subscription.

This study assumes that the competitive state of these markets adheres to the original conditions of the LVC model. The shared environment under which mobile and fixed-line phones are competing is the United States, European Union voice broadband industry. The constrained resources are potential consumers. As the U.S possess a wide range of telephone companies and mobile cellular providers, it would be complicated to consider each individual company. Also, as there has been a rapid change in the growth pattern of these two technologies during the selected timeframe, analyzing on a company level might lead to misleading results about the macro shifts the industry. Hence, the data would be the total yearly subscription of mobile cellular and land lines.

c. Parameter estimation

The Lotka-Volterra model cannot be used to generate a demand function for mobile and fixed phones unless if the currently unavailable parameters $\alpha, \beta, and\ \gamma$ are estimated. This task was managed by applying the Gauss-Newton non-linear least squares method provided in the software EViews 10.0. The method uses the Marquardt algorithm, an interative procedure that depends on the specification of the convergence criterion and the maximum number of iterations. The convergence criterion in this study is set to 0.001, meaning that the iterations would stop when the maximum of the percentage changes in the coefficients is less than 0.1%.

The equations' input for EViews take the forms of:

X = C(1)*X(-1)/(1+C(2)*X(-1)+C(3)*Y(-1))

Y = C(1)*Y(-1)/(1+C(2)*Y(-1)+C(3)*X(-1))

where X,Y on the left hand side represent the dependent variables – the cumulative annual demand for fixed-line phones (X) and mobile phones (Y). *C(1 to 3)* are the parameters $\alpha, \beta, and\ \gamma$ respectively. *X(-1)* and *Y(-1)* are the total number of subscription for the preceding year.

The hypotheses for each case could be re-written as follows:

- H0: γ, the interaction parameter, does not influence a species' diffusion *(p>0.05)*

- H1: γ influences a species' diffusion *(p<0.05)*

Failure to reject H0 would mean that the effect of one species interfering with the other's growth is non-significant. In contrast, rejecting H0 would mean that the interaction between these species influence growth.

2. Forecasts and accuracy

a. United States (U.S)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.077412	0.029193	36.90658	0.0000
C(2)	2.64E-10	1.75E-10	1.510980	0.1413
C(3)	2.54E-10	3.67E-11	6.920918	0.0000

R-squared	0.982434
Adjusted R-squared	0.981262
S.E. of regression	3314698.
Sum squared resid	3.30E+14

Table 1: Estimation results of the LVC model on fixed-line telephones (U.S)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.357971	1.785867	1.880303	0.0698
C(2)	3.33E-09	2.38E-09	1.402833	0.1709
C(3)	8.10E-09	6.76E-09	1.198895	0.2400

R-squared	0.997717
Adjusted R-squared	0.997565
S.E. of regression	6790913.
Sum squared resid	1.38E+15

Table 2: Estimation results for the LVC model on mobile phones (U.S)

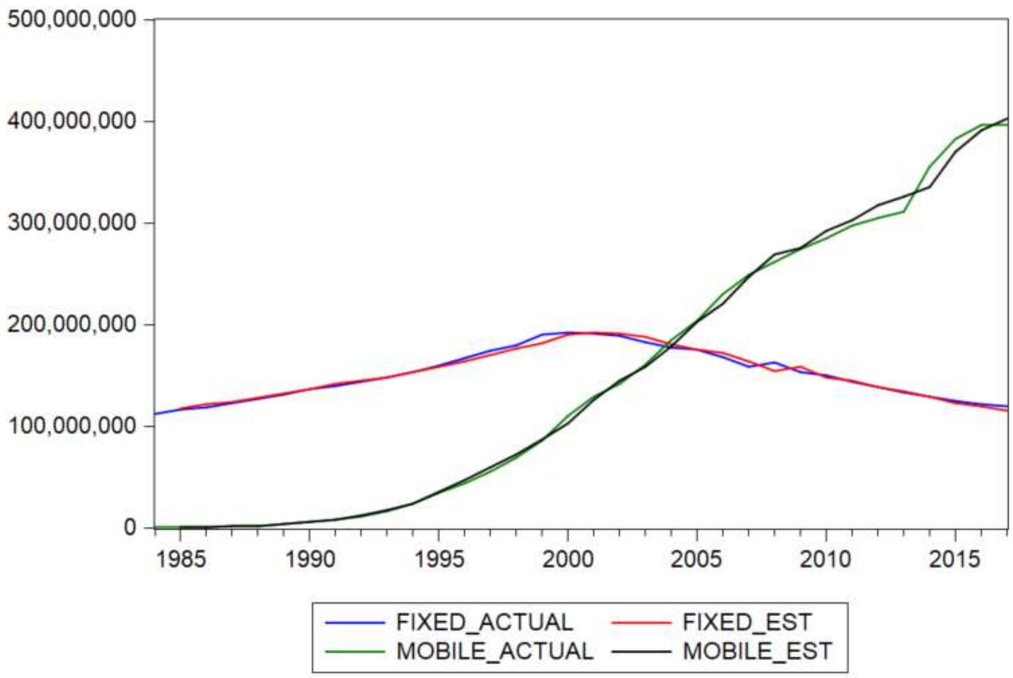


Figure 3: Estimated data and actual data (U.S)

Mobile phones	MAE	4239702
	MAPE	5.326940
Fixed-line telephones	MAE	2404771.
	MAPE	1.553689

Table 3: MAE and MAPE for the Lotka-Volterra model (U.S)

b. European Union (EU)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.066869	0.016952	62.93568	0.0000
C(2)	1.35E-10	8.63E-11	1.568258	0.1261
C(3)	8.71E-11	1.11E-11	7.848388	0.0000

R-squared	0.996135
Adjusted R-squared	0.995907
S.E. of regression	2686171.
Sum squared resid	2.45E+14

Table 4: Estimation results of the LVC model on fixed-line telephones (EU)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2.942868	1.536684	1.915077	0.0639
C(2)	1.81E-09	1.06E-09	1.709494	0.0965
C(3)	3.62E-09	3.92E-09	0.923312	0.3624

R-squared	0.997877
Adjusted R-squared	0.997752
S.E. of regression	12874860
Sum squared resid	5.64E+15

Table 5: Estimation results of the LVC model on mobile phones (EU)

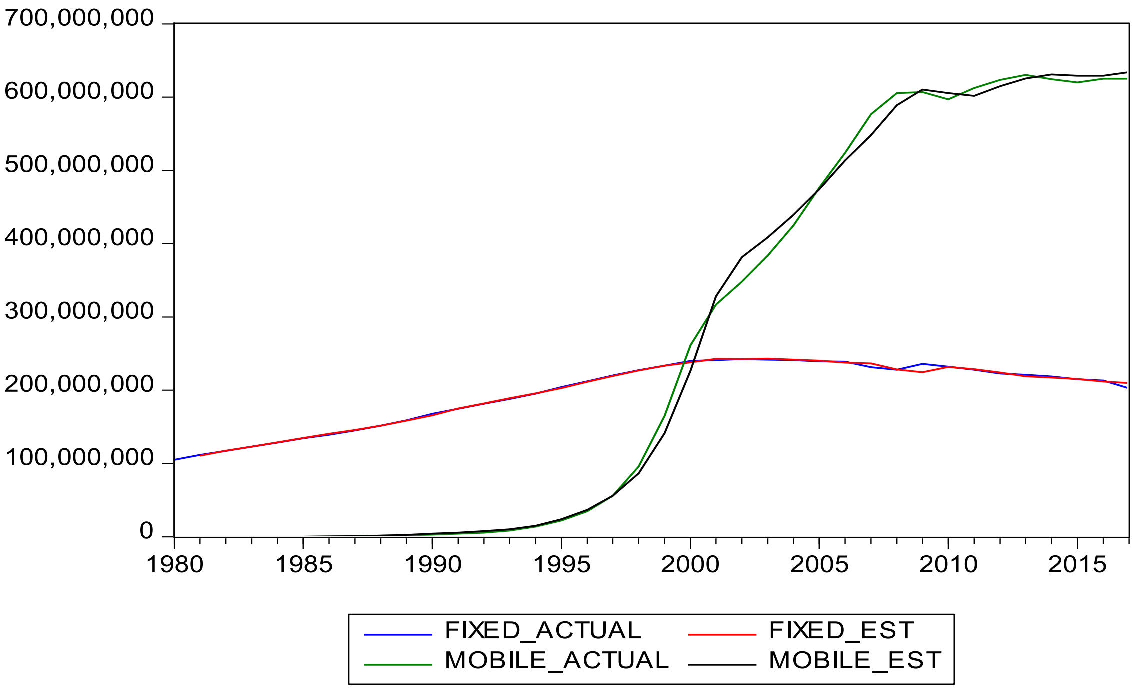


Figure 4: Estimated data and actual data (EU)

Mobile phones	MAE	7686501.
	MAPE	11.29533
Fixed-line telephones	MAE	1425297.
	MAPE	0.690319

Table 6: MAE and MAPE for the Lotka-Volterra model (EU)

c. China

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.478032	0.059946	24.65617	0.0000
C(2)	9.22E-10	1.78E-10	5.170359	0.0000
C(3)	3.01E-10	3.51E-11	8.558603	0.0000

R-squared	0.995059
Adjusted R-squared	0.994693
S.E. of regression	9350513.
Sum squared resid	2.36E+15

Table 7: Estimation results of the LVC model on fixed-line telephones (China)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.687077	0.159631	10.56861	0.0000
C(2)	3.21E-10	6.24E-11	5.147155	0.0000
C(3)	7.52E-10	3.13E-10	2.405844	0.0233

R-squared	0.998750
Adjusted R-squared	0.998657
S.E. of regression	18771975
Sum squared resid	9.51E+15

Table 8: Estimated results of the LVC model on mobile phones (China)

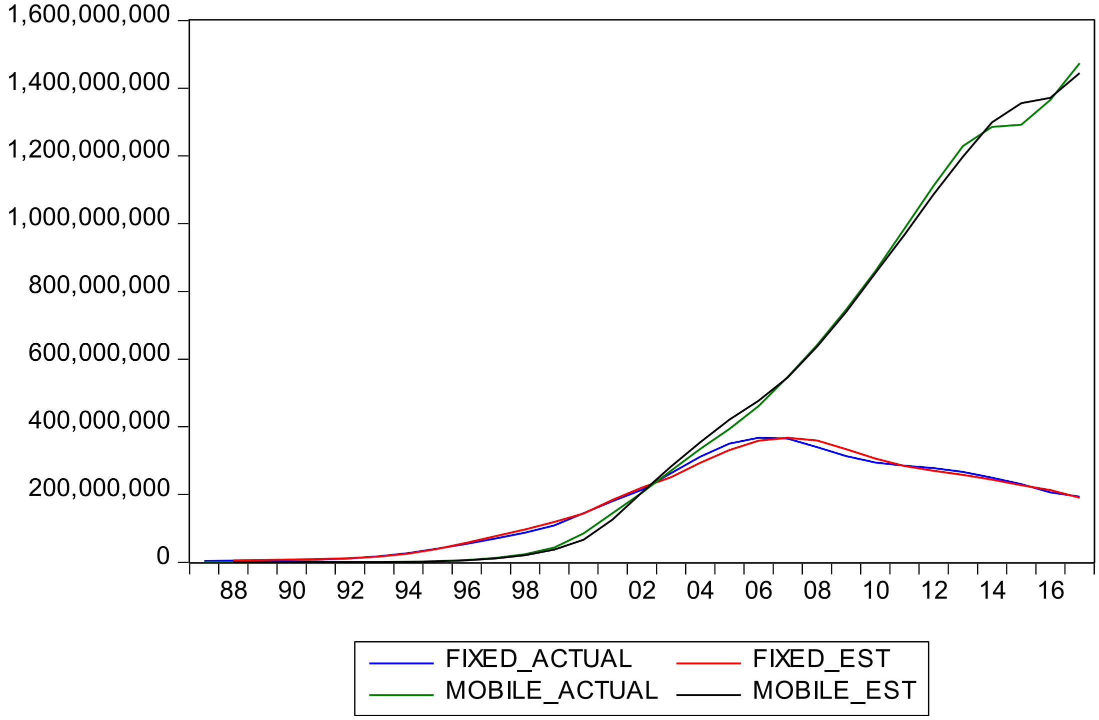


Figure 5: Estimated data and actual data (China)

Mobile phones	MAE	11169763
	MAPE	15.04703
Fixed-line telephones	MAE	6540709.
	MAPE	6.587178

Table 9: MAE and MAPE for the Lotka-Volterra model (China)

The forecast periods for the US, the EU, and China are 1985-2017, 1981-2017, and 1988-2017 respectively as there is a lag of 1 year in each case. The regressions' R-squared values are well above 0.95 for all tests, indicating a good fitting performance. This could also be seen from the fitted graphs as the estimated figures seem close to the actuals.

In all three cases, the MAPE values seem to be better for fixed-line phones than for mobile phones. Compared to existing literature, the MAPEs found in this study is relatively higher as many previous authors have been able to maintain it at a level less than around 5%

(See for example: Lee, et al., 2005; Kim, et al., 2006; Tsai, 2017; Gupta & Jain, 2016). In fact, none of the MAPEs is lower than 5% for any forecast of mobile phones. Thus, this indicates that out model, while is producing close estimations for the telephone market, has some errors for its predictions of mobile phone demand. To gain a closer understanding of why these values are unfavorable, it is of importance that the trends for the standard errors of both market be investigated. Separated forecasts with standard errors for mobile phones and telephones in China have been chosen for this purpose. It is noticeable that the MAE values for fixed-line phone forecasts are lower than those of mobile phones. This indicates that this market has less forecast inaccuracy than the other one.

It should be noted that in an attempt to reduce compounded errors over time, this study has performed all the forecasts with a static method. As this method uses the actual value of each independent variable as an input to produce each estimated value, the standard errors are expected to appear more controlled and stable.

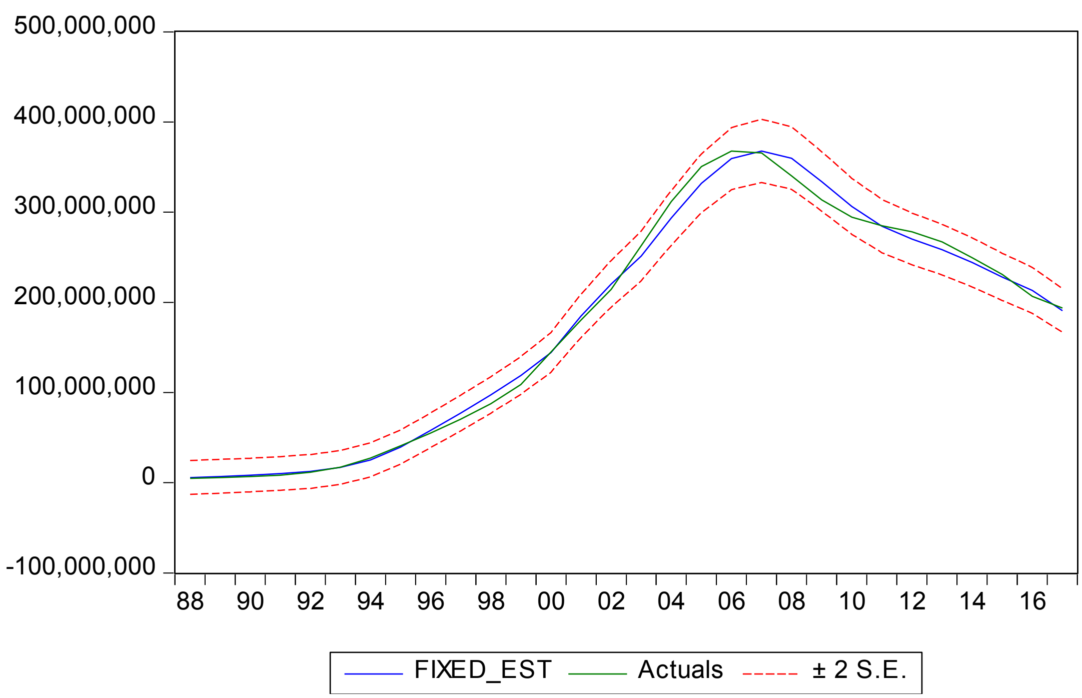


Figure 6: Confidence error bands for fixed-line phones in China

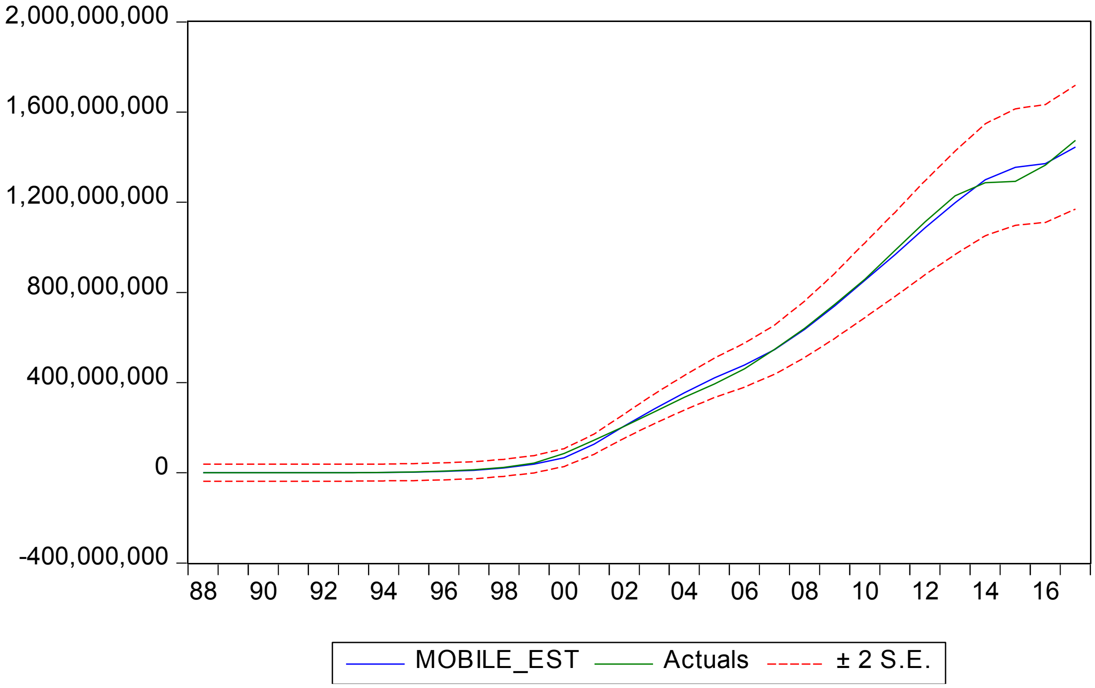


Figure 7: Confidence error bands for mobile phones in China

Based on figures 6 and 7, it is clear that the standard error distribution for the telephone market is close to our expectations for a static forecast. However, the confidence bands for mobile phones seem to widen over time starting from the inflection point at around the year 1999 or 2000. This trend would be more reasonable for a dynamic forecast where the predicted value for year Y would be used to produce the next forecast for year Y+1. A valid explanation for this issue could be the number of observations in the sample. A range of approximately 30 observations is quite insufficient for annual forecasting. If there had been more actual data points, the error terms could have been reduced and, thus, the MAPEs would have been smaller, indicating less forecast uncertainty and inaccuracy. Moreover, the expansion of confidence bands from the inflection point could be due to the growth in annual change in demand. Prior to 1999, subscription for mobile phones in China seem to be growing at an extremely slow pace which lead to a rather flat trend line. During this period, the estimated data appears to fit neatly with the actual data. However, from 1999 to 2017, the change in demand rises sharply, creating a steep upward curve. Consequently, the fitted curve became more dissimilar to the actual one. The complexity of using annual data combined with these rapid increases might have greatly decreased the accuracy of the estimated observations as we forecast longer into the future.

b. Competitive relationship interpretation

Leslie (1957) derived the relationships between the coefficients a_i, b_i, and c_i of the non-discrete Lotka-Volterra model (Eqs. (5) and (6)) and those of the difference equations (7) and (8) as follows:

$$a_i = \ln \alpha_i$$

$$b_i = \frac{\beta_i a_i}{\alpha_i - 1} = \frac{\beta_i \ln \alpha_i}{\alpha_i - 1}$$

$$c_i = \gamma_i \frac{b_i}{\beta_i} = \frac{\gamma_i}{\beta_i} \frac{\beta_i a_i}{\alpha_i - 1} = \frac{\gamma_i \ln \alpha_i}{\alpha_i - 1}$$

If $\alpha_i > 0, \alpha_i \neq 1$, $\frac{\ln \alpha_i}{\alpha_i - 1}$ should always be positive. Thus, the sign of γ_i must be the same as the sign of c_i. Therefore, the type of competitive roles could be interpreted with the sign of γ_i. Modis (1999) classified 6 types of competitive relationships based on c_i, which has been translated into γ_i in the following table:

γ_1	γ_2	***Type***	***Explanation***
+	+	Pure competition	Both species suffer from each other's existence
-	+	Predator – prey	One is food for the other species which relies on it to survive
-	-	Mutualism	A win-win situation in which both species benefit from each other's existence, or symbiosis
-	0	Commensalism	A parasitic type of relationship in which one benefits from the other that is unaffected by this interaction
+	0	Amensalism	One suffers from the existence of the other that remains unaffected
0	0	Neutralism	No interaction whatsoever between both species

Table 10: Classification of competitive relationships

Referring to tables 1 and 2, in the case of the US, it is clear that both γ_1 and γ_2 carry a positive sign, suggesting that the mobile phone market and the telephone market are under pure competition. Note that, however, because the t-value for γ_2 is not statistically significant (p=0.24 > 0.05), this would indicate that the change in growth of mobile subscription is not affected by the change in growth of telephone subscription. Also, γ_2, although is higher than γ_1 is very close to 0 (0.0000000081). Therefore, it is reasonable to assume that $\gamma_2 = 0$. The same cannot be said for γ_1 as the statistically significant p-value = 0 < 0.05 indicates that the diffusion of mobile phones does influence that of fixed land lines. Due to these reasons, the results imply that these markets are share an amensalism dynamic with mobile phones being unmoved by the growth fluctuations of telephones, and telephones being inhibited by the existence of mobile phones.

In the same manner, γ_1 and γ_2 in the case of the EU also carry positive signs. However, since only γ_1 is statistically significant (p = 0.000 < 0.05), it is reasonable to conclude that γ_2 should be 0. This suggests that the relationship between mobile phones and telephones could be classified as amensalism. The interference from the growth of mobile phones on that of land lines (γ_1) is significant, meaning that the latter suffers from the existence of the former. Nevertheless, the reverse could not be said since γ_2 has a p-value that is greater than 0.05, indicating non-significance and a failure to reject the null hypothesis.

In the case of China, at the 95% confidence level, all parameters seem to be statistically significant (p<0.05). Since the signs of γ_1 and γ_2 are positive, we could infer that the mobile phone market and the telephone market in China are in pure competition. Interestingly, results from China's parameters also shows that C(2) is statistically significant in both cases. This implies that the niche capacity of each species does impact their diffusion, indicating high competition and 'self-squeezing' effects among individuals of each species. Perhaps, due to resource constraints in this environment, the 2 groups must minimize its resource usage. In reality, this could be translated as the competition among Chinese firms in each market is so high that they may be strategizing by cutting cost, using lean operation models and so on. Contrary to the US and the EU where one is threatened by the growth of the other, both species suffer from coexistence in this case.

3. Equilibrium analysis

Having inferred the competitive roles of the selected markets, we could now attempt to produce an equilibrium analysis to gauge whether or not both markets could still expand. In equilibrium, there is no change over time for each species. Hence equations (5) and (6) must be equal to zero, as follows:

$$\frac{dX}{dt} = a_1X - b_1X^2 + c_1XY = X(a_1 - b_1X - c_1Y) = 0 \quad (13)$$

$$\frac{dY}{dt} = a_2Y - b_2Y^2 + c_2YX = Y(a_2 - b_2Y - c_2X) = 0 \quad (14)$$

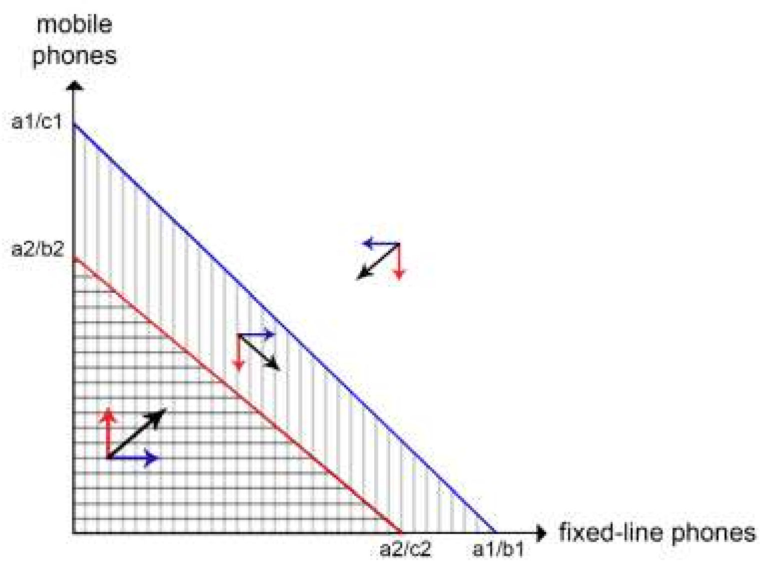
This would lead to two cases. Firstly, there is an equilibrium where X=0 and Y=0 at which both species do not exist. This is clearly explained by the fact that at the point of extinction, there is no growth factor thus both species will stay at this point. However, this case is highly impractical. Alternatively, an equilibrium could be captured at the point where

$$\begin{cases} a_1 - b_1X - c_1Y = 0 \\ a_2 - b_2Y - c_2X = 0 \end{cases}$$

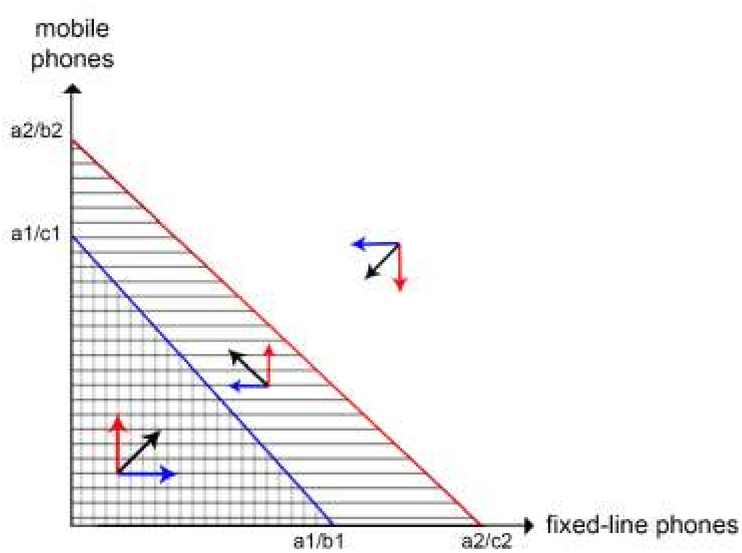
$$\Leftrightarrow \begin{cases} X = \dfrac{a_1 - c_1Y}{b_1} \\ Y = \dfrac{a_2 - c_2X}{b_2} \end{cases} \quad (15)$$

If $X < \frac{a_1 - c_1Y}{b_1}$ then *dx/dt > 0.* Then, there would be an increase in the number of landline telephones. If $X > \frac{a_1 - c_1Y}{b_1}$ then *dx/dt<0*. This would mean that there would be a reduction in the number of telephones. Likewise, for the number of cellular subscriptions, the same could be said when $Y < \frac{a_2 - c_2X}{b_2}$ and $Y > \frac{a_2 - c_2X}{b_2}$.

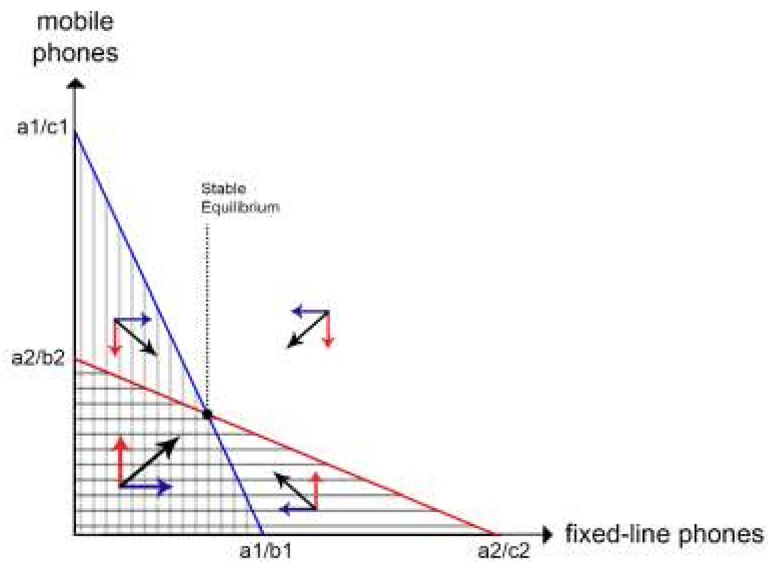
If there is an intersection of the X and Y of equation set (15) in the first quadrant, meaning that *dx/dt* and *dy/dt* cross, there would be an equilibrium. If there is not an intersection in the first quadrant, then an equilibrium would not exist and there would be a case of one species being domineering. The functions in the equation set (15) represent the linear isoclines of both species. At X = 0, Y = a_1/c_1 for species 1 (mobile phones) and X = a_2/b_2 for species 2 (telephones). At Y = 0, X = a_1/b_1 and X = a_2/c_2 for species 1 and 2, respectively. Consequently, we could derive four possible cases as follows:



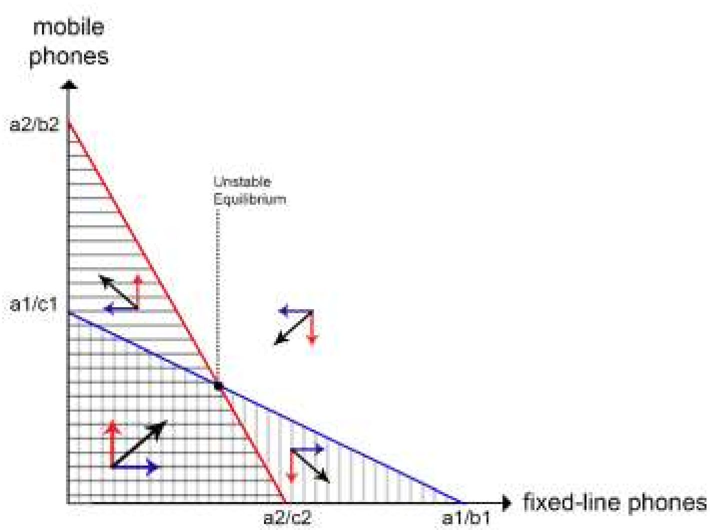
a) Telephone market wins



b) Mobile phone market wins



c) Stable Equilibrium



d) Unstable Equilibrium

Figure 8: possible interactive outcomes for 2 species in the same environment

The vertically marked area represents the region under the blue line dX/dt = 0, or $X = \frac{a_1 - c_1 Y}{b_1}$, in which total subscription for fixed-line phones will grow. Similarly, the horizontal hatching under the red line dY/dt = 0 covers the area in which total subscription for mobile phones would grow. The region with the crisscross pattern is the overlapping part of the growth functions where both species could increase in size. The blue vector is the movement of species 1, also known as X or the land-line phone market. Similarly, the red vector is for that of species 2, Y, or the mobile phone market. The black vector is the overall effect created from the movements of both species. Each point on the graph represents combinations of subscription levels for mobile phones and fixed-line phones.

For the first 2 scenarios in which one species' growth function lies above that of the other, the one with a lower growth capacity would be crowded out and dominated. This is because the stronger species' size can increase to a level above that of the weaker species. Consequently, it is likely that the domineering species would replace the weaker one completely. As the growth functions do not cross, an equilibrium does not exist in both cases.

In the last two cases, the lines dX/dt and dY/dt do cross. Thus, an equilibrium point can be found. By equating the growth functions, we could yield the equilibrium values of $X^* = (a_1b_2 - a_2c_1)/(b_1b_2 - c_1c_2)$ and $Y^* = (a_2b_1 - a_1c_2)/(b_1b_2 - c_1c_2)$. It should also be noted that an equilibrium should only be valid if it is in the first quadrant where X and Y is greater than or equal to 0. In the first 2 cases, the isoclines technically intersect each other, but in a different quadrant where X, Y or both of them would be negative. The case of a negative growth is unrealistic and impractical. Thus, to perform equilibrium analyses, only the first quadrant where X and Y are equal to or greater than 0 should be examined.

To clearly depict the outcome in each case and gauge whether or not an equilibrium is stable, meaning that it is a point at which both species are not inclined to increase in size, it is necessary to investigate each species' behavior within the regions that are divided by the intersections of the isoclines.

Regarding case (a), if the initial state lies under the red line (dY/dt = 0), then both species have not reached maximum growth. Hence, species 1, or X, would move to the left as it tries to attain higher population size. Likewise, species 2, or Y, would move upward. The average overall effect is represented by the black vector that is stretching away from the origin. However, if the initial state is above species 2's growth function and under specie

1's growth function, Y would shift downward, essentially decreasing in size, and X would shift to the right. The total effect is, thus, approaching the maximum population size for telephones that is equal to a1/b1. Therefore, when the growth capacity of species 1 (X) is greater than that of species 2 (Y), the former will keep reach its population size (a1/b1) while the latter will be driven to extinction. If the starting values for X and Y are above both isoclines, the environment simply does not have sufficient resources to support the growth of both species and, thus, force them to depopulate. When they enter the capacity region of species 1, specie 1 will crowd out species 2 and as it re-optimizes to a1/b1. In the same pattern, in case (b), the mobile phone market, or species 2, would dominate the telephone market, or species 1. The outcome would result in extinction for species 1 and maximum growth for species 2 at the population level of a2/b2.

When there is an intersection in the first quadrant, we could consider two scenarios where there could be a stable equilibrium or an unstable equilibrium. With a stable equilibrium, $a_2/b_2<a_1/c_1$ and $a_2/c_2>a_1/b_1$. The markets will gradually approach X* and Y*, regardless of their original conditions. Consequently, in the vertically marked region, dX/dt>=0 and dY/dt =<0 so X=<X* and Y>=Y*. Likewise, in the horizontally marked region, dX/dt =<0 and dY/dt >=0 so X >=X* and Y=<Y*. In contrast, if $a_2/b_2>a_1/c_1$ and $a_2/c_2<a_1/b_1$, the equilibrium would be unstable.

To be more specific, in case (c), for points in the crisscross section or above the growth functions, species would re-optimize by increasing or decreasing their size, respectively. However, this time they would also be inclined to approach the equilibrium where X cannot increase by moving to the left and Y cannot increase by moving upward. Both X and Y would not be incentivized to decrease by moving in their opposite direction as the equilibrium point maximizes population and exhausts all environmental resources. These movements are also illustrated by the total effect arrows that are approaching the equilibrium. If the population levels for both species are in the vertically hatched section where they are above species 2's growth function but under species 1's growth function, X will shift to the left to approach maximum population size and Y will decline until the environment and support its size. Once again, the cumulative effect of these behaviors would incline toward the equilibrium. The same pattern could be captured from all points in the horizontally hatched area where Y has incentives to increase and X has to decrease. Therefore, it could be concluded that regardless of the initial state of both markets, they will converge to a stable equilibrium point as long as $a_2/b_2<a_1/c_1$ and $a_2/c_2>a_1/b_1$.

In the fourth case where there is an intersection between the isoclines, an equilibrium also exists. However, contrary to the previous case, this one is an unstable equilibrium. For any point that lies wholly above or below the isoclines, the species will increase or decrease in size as in the other 3 cases. For points in the horizontally hatched area, species 2 will increase to reach its maximum capacity by having Y move upward, while species 1 will decrease as it is overpopulated. These behaviors, thus, create a total effect vector that approaches the Y axis. In a like manner, if the markets initially exist in the vertically hatched area, Y-values will decline and X-values will rise, leading to a total effect vector that approaches the X axis. Therefore, it could be said that in these two circumstances, the outcome is similar to that of case (a) or (b) where competitive exclusion eradicates one species, while the other species with a higher growth capacity will dominate the shared space and win. As it is uncertain whether the species will depart from this equilibrium, it is considered unstable.

a. United States

To graphically illustrate the interactive outcome, the discrete parameters $\alpha, \beta,$ and γ need to be converted to their continuous versions a, b, and c. This could be done by using Leslie's relationship equations:

$$a_1 = \ln \alpha_1 = \ln(1.077412) = 0.074562$$

$$b_1 = \frac{\beta_1 a_1}{\alpha_1 - 1} = \frac{\beta_1 \ln \alpha_1}{\alpha_1 - 1} = \frac{(2.64E-10) \times 0.074562}{1.077412 - 1} = 2.54E - 10$$

$$c_1 = \gamma_1 \frac{b_1}{\beta_1} = (2.54E - 10) \times \frac{2.54E - 10}{2.64E - 10} = 2.45E - 10$$

Applying the same operations to species 2, the mobile phone market, would yield the following results:

$a_2 = 1.211337\ ;\ b_2 = 1.71E - 09\ ;\ c_2 = 4.16E - 09$

Thus, the axis intercepts could be derived as follows:

a1/b1	2.93E+08
a2/c2	2.91E+08
a2/b2	7.08E+08
a1/c1	3.05E+08

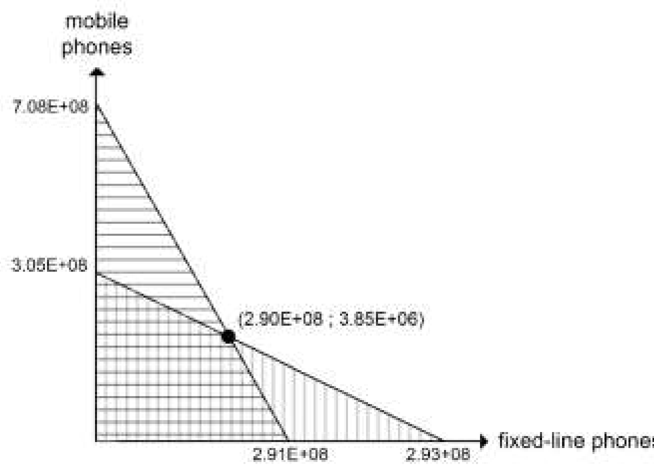


Figure 9: First-quadrant graph of the interactive outcome for the United States

Because $a_2/b_2 > a_1/c_1$ and $a_2/c_2 < a_1/b_1$, the equilibrium in this case is unstable, meaning that the outcome is uncertain. The equilibrium values can be computed as $X^* = (a_1b_2 - a_2c_1)/(b_1b_2 - c_1c_2) = 2.90E+08$ and $Y^* = (a_2b_1 - a_1c_2)/(b_1b_2 - c_1c_2) = 3.85E+08$.

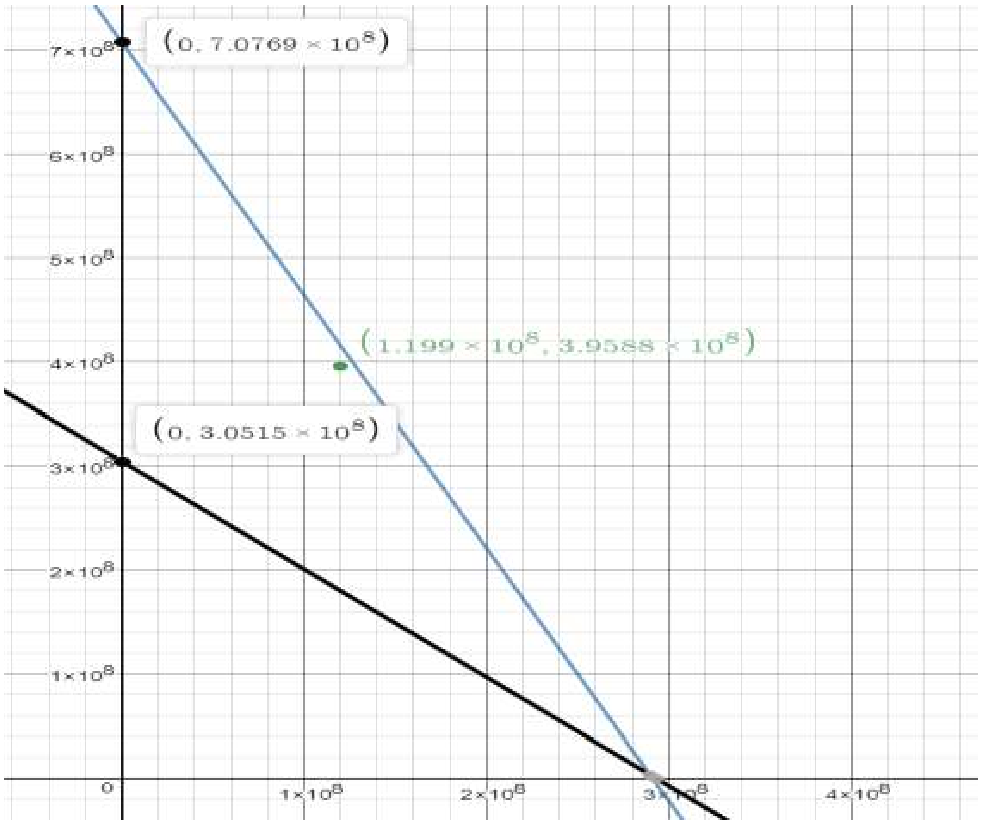


Figure 10a: The current state and Y-intercepts of the growth functions (US)

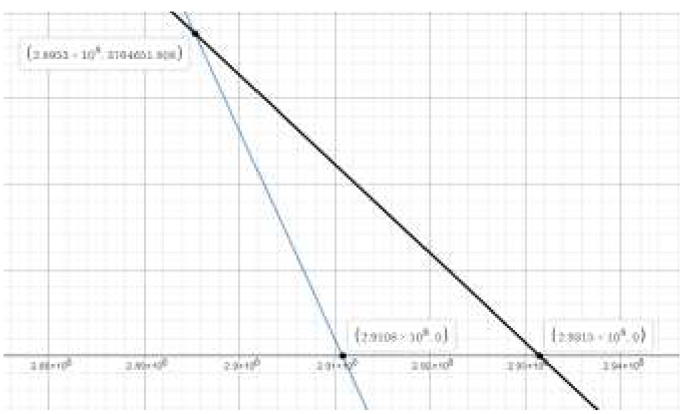


Figure 10b: The equilibrium point and X-intercepts for the growth functions (US)

As of 2017, the total subscription for mobile phones is Y = 395881000 and for telephones is X = 119902000. This indicates that they are in the horizontally hatched region where all points will move away from the equilibrium and toward the maximum growth level of species 2 which is at a2/b2 = 7.08E+08 (Figure 10a). In this case, there is competitive exclusion where the species whose population is within its growing capacity will increase and dominate the other one that has to constantly re-optimize to get back on its growth function. Thus, species X – the telephone market will have to decrease until its size is equal to 0. From figure 10a, it is clear that the current state is very close to the growing capacity of the mobile phone market. Thus, it is expected that the annual rise in subscription for this market will slow down. For the fixed-line phone market, however, subscription should continue declining at a higher rate than. The diffusion patterns in Figure 3 seem to support this claim as the actual data curve for mobile phones from 2015 to 2017 has become almost flatten, suggesting stagnant growth. Oppositely, the trend line for telephone diffusion is clearly downward sloping with a steeper slope. In fact, it could also be seen that from around the year 2000, the inflection point, to 2017, land line usage has dropped to approximately the same amount that was recorded in 1984.

b. European Union

Applying the same operations with Leslie's discrete to continuous equations, the following parameters are obtained:

$a_1 = 0.064728\ ;\ b_1 = 1.31E - 10\ ;\ c_1 = 8.43E - 10$

$a_2 = 1.079385\ ;\ b_2 = 1.01E - 09\ ;\ c_2 = 2.01E - 09$

From these values, we could compute the axis intercepts:

a1/c1	7.68E+08
a2/b2	1.07E+09
a2/c2	5.37E+08
a1/b1	4.95E+08

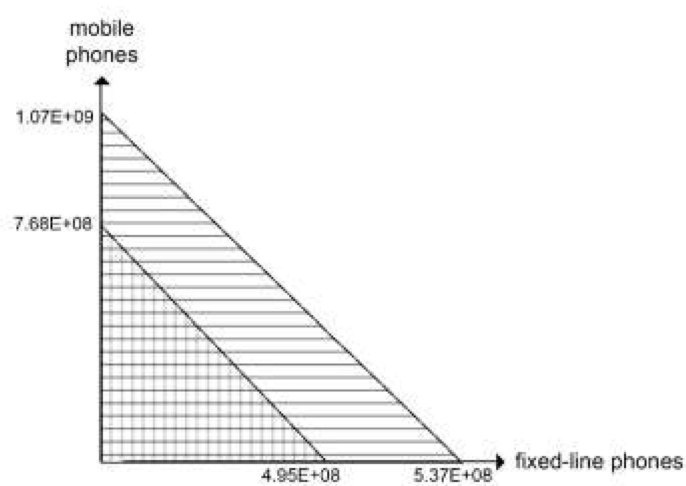


Figure 11: First-quadrant graph of the interactive outcome for the European Union

In this scenario, since a2/b2 > a1/c1 and a2/c2 > a1/b1, the isocline of the second species lies wholly above that of the first species. This means that the growth capacity of the mobile telephone market surpasses that of the fixed-line phone market. Thus, as time passes, traditional telephones could be replaced completely by mobile phones. The eventual outcome would be for mobile phones to reach its maximum size at a_2/b_2 = 1.07E+09 while fixed-line phones would become obsolete.

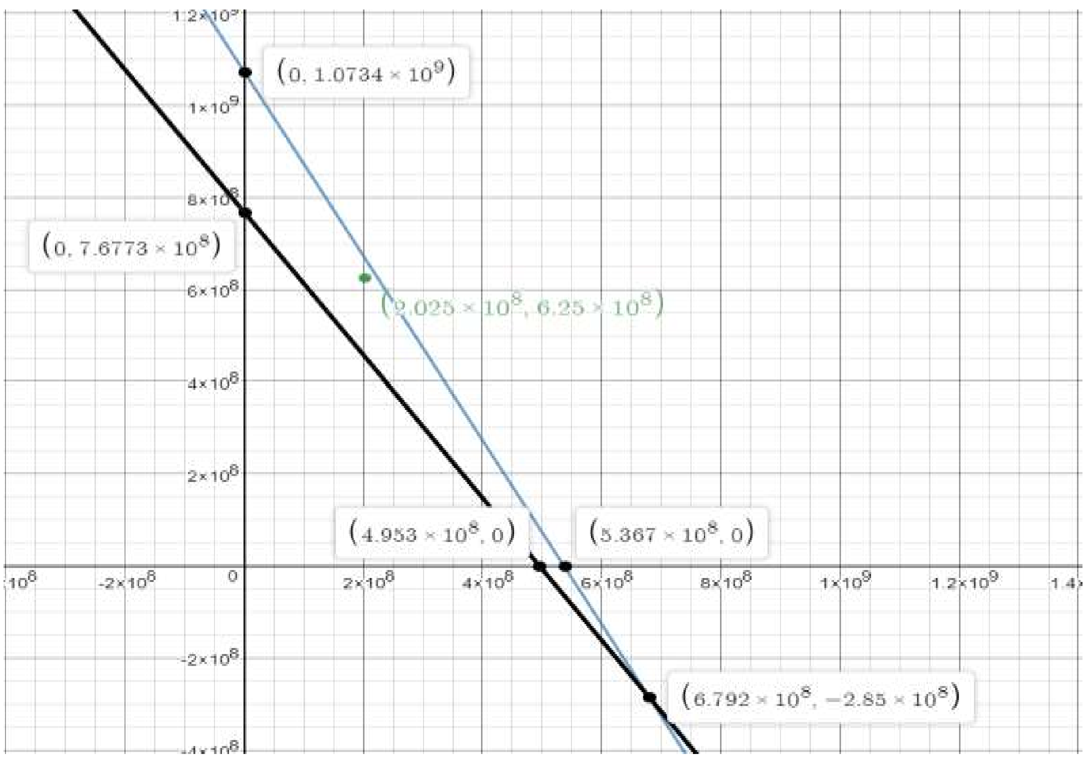


Figure 12: Actual graph for the growth functions in the four quadrants (EU)

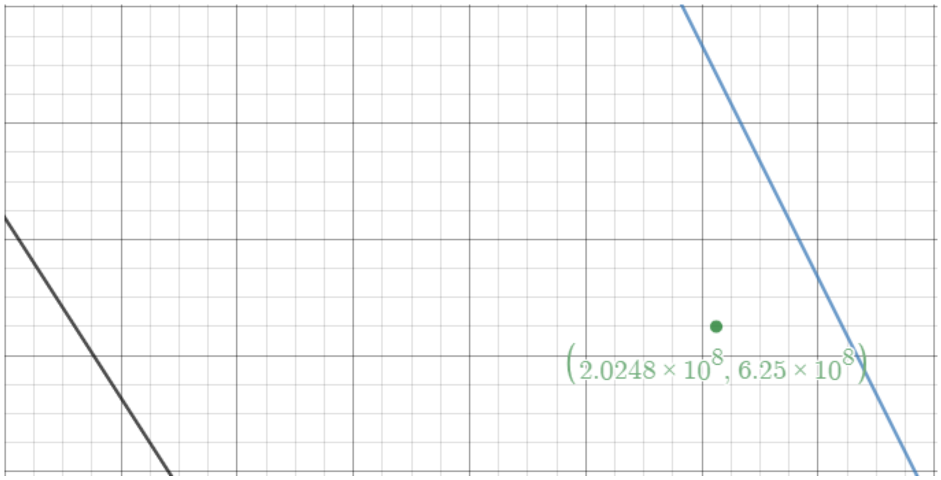


Figure 13: The current state (EU)

As of 2017, the number of mobile cellular users in EU is 625000799 while that of telephone users is 202478401 (Figure 12). These numbers suggest that these markets are in the horizontally hatched region where it is likely that X will decrease and Y will increase. However, since this point is already quite close to the isocline for mobile phones, the size of this market will not increase by much, as opposed to that of telephones which should be reduced at a faster pace over time (Figure 13). Referring to Figure 4, we could see that this finding is supported by the trend lines of the actual data. The curve for mobile phone diffusion appears upward sloping and flat, indicating slow growth. In contrast, the curve for telephone subscription is downward sloping. Currently, it is unclear whether the absolute change in growth for land lines would be more than that of mobile phones. Eventually, in the EU, telephone and mobile subscription should converge toward 0 and 1.07E+09, respectively.

c. China

Again, using Leslie's method of transforming the parameters from discrete to continuous mode, we could yield the following values:

$a_1 = 0.390711\ ;\ b_1 = 7.54E - 10\ ;\ c_1 = 2.46E - 10$

$a_2 = 0.522997\ ;\ b_2 = 2.44E - 10\ ;\ c_2 = 5.72E - 10$

Consequently, the values for the axis intercepts are:

a1/c1	1.59E+09
a2/b2	2.14E+09
a2/c2	9.14E+08
a1/b1	5.18E+08

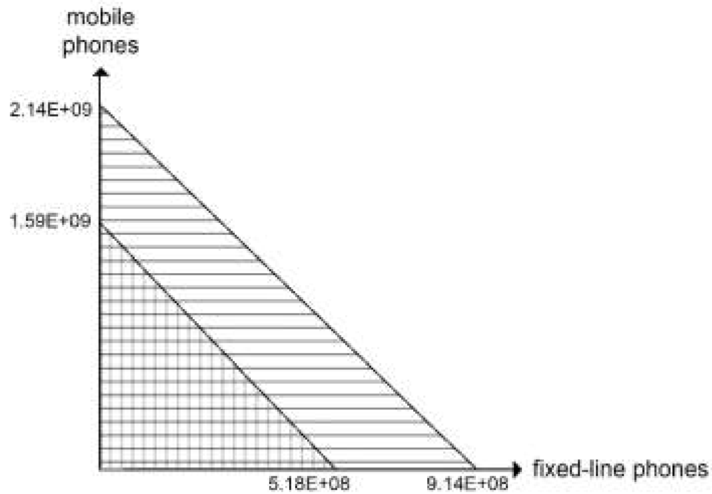


Figure 14: First-quadrant graph of the interactive outcome for China

Similar to EU's interactive outcome graph, this one depicts that species 2's growth function is above that of species 1, indicating greater population capacity with respect to limited environmental resources. Thus, an equilibrium does not exist in this case. Also, species 1 which is the telephone market will be dominated by species 2 which is the mobile phone market.

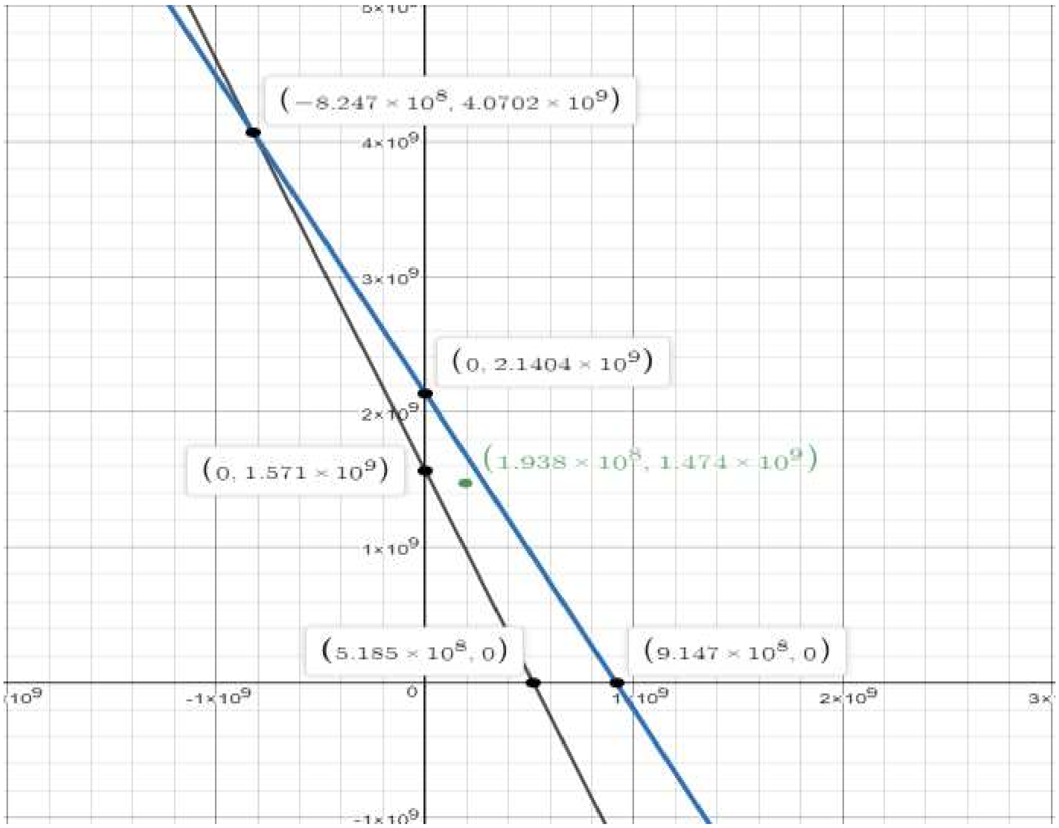


Figure 14: Actual graph for the growth functions in the four quadrants (China)

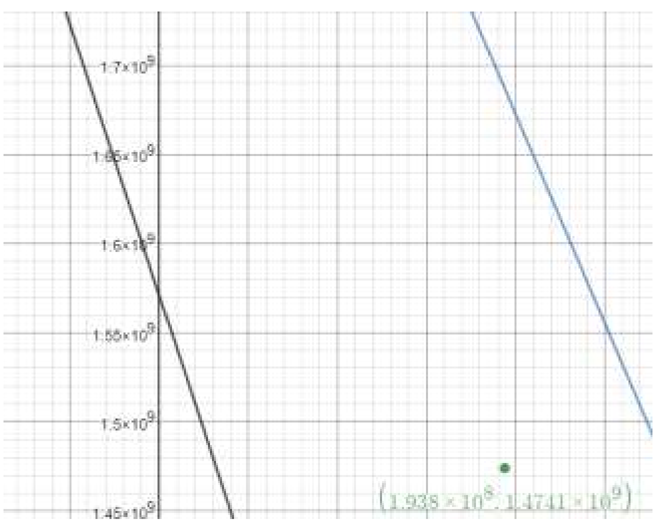


Figure 15: The current state (China)

As of 2017, Chia's mobile subscription is at Y = 1474097000 and telephone subscription is at X = 193762000 (Figure 14). Hence, the data indicates that their current state is in the horizontally hatched region where X would be inclined to decrease and Y would be inclined to increase. As the gap from $X_{current}$ to the line dX/dt = 0 is slightly narrower than the gap from the gap from $Y_{current}$ to dY/dt = 0 (Figure 15), we could infer that the increase in mobile subscription should be moderately higher than the reduction in land line subscription. Revisiting the actual data plot for China (Figure 5), it is clear that this statement aligns with recent trends as the growth curve of mobile phones is relatively steeper than that of telephones. As time passes, the current state would be approaching the mobile phone market's Y-intercept, which is equal to 2.14E+09. Eventually, mobile phones would dominate the voice telephony sector and render land lines obsolete.

D. Discussion

In this paper, we analyzed the competitive relationship between the mobile phone market and the telephone market in the top three economic aggregates with the highest GDP_{pc} as of January 2019 – the United States, the European Union, and China. In general, the model is quite promising as it produced estimates that are close to actual observations. In addition, the theoretical suggestions of the behaviors of both markets are validated by real diffusion patterns.

The reliability and accuracy analysis shows that the Lotka – Volterra model performs well with a good fit well above 0.95 for every case. This result is similar to previous literature that proved that the model indeed can explain the majority of the observations (Wulf, et al., 2013; Tseng, et al., 2014; Wang & Wang, 2016). However, the MAPE values for forecasts of the telephone market are higher than those that have been found before. The issue with the MAPE values could be alleviated by using a different data set that is larger and more frequently recorded.

In terms of the competitive relationship analysis, it was concluded that the interaction types, based on the γ values, are amensalism, amensalism, and pure competition for the US, the EU, and China, respectively. Interestingly, it should be noted that China is the economy with the highest mobile telephony distribution, up to 1 474 097 000 in 2017, while that the US and EU is more than half less at 395 881 000 and 625 000 799. China is also holding the highest GDP_{pc} at 27,449,046 USD as of October 2018 (International

Monetary Fund, 2018). In all regions, the data implies that mobile phones will replace telephones completely in the long run and approach their maximum population.

Previous tests of the same Lotka-Volterra model not just with fixed-to-mobile substitution but also with a wide range of products and technology have discovered interrelationship types that explain well the competitive situation. For example, the first study of the model on mobile diffusion found that PCS and mobile cellular were commensalisms in Korea. This was in accordance with their actual behaviors as the market share increased with but never surpassed that of mobile cellular, even when the government temporarily suspended cellular service (Kim, et al., 2006). Many previous papers have also successfully inferred interrelationship types, including commensalism (Kim et al., 2006; Wulf, et al., 2013; Gupta & Jain, 2016), amensalism (Avila, et al., 2018), and predator – prey (Kreng & Wang, 2010; Wang & Wang, 2016; Tsai, 2017).

To extend the discussion on the results for China, which were outstanding from the rest, there might be several reasons for why the markets are currently in a pure competition state. As China is a densely populated country with a highly uneven distribution of wealth in rural versus urban areas, the spread of mobile phones might suffer from the existence of telephones because the many low-income Chinese households might prefer land lines as opposed to cellular services. As prices are usually a lot cheaper for telephones, low-income users might not opt for the other more advanced alternative. Thus, as long as the wealth gap effects remain dramatically different for each part of the country, these markets might not leave this competitive state easily.

Revisitng the original hypotheses, which are at the end of the literature review, and the equilibrium analysis, ti is clear that H2 can be approved. All of the interactive outcome maps suggest that mobile phones will increase and render telephones obsolete. The first hypothesis, H1, on how the interference of mobile phones on telephones has also been confirmed with the significant t-stat values for γ_1.

E. Limitations and further research

1. On the model's performance

The most major limitation of this research is complications with the selected data. In the accuracy and reliability analysis, it was shown that the limited range of observations, combined with lags in the forecasts, might have increased the errors, consequently enlarging MAPE values and widening the confidence bands over time. The MAPE values for the telephone market's forecast were especially higher than those of existing studies that are under 5% (See for example: Lee, et al., 2005; Kim, et al., 2006; Tsai, 2017; Gupta & Jain, 2016). However, there have also been authors who had MAPE values above 5%, even at around 20% (See for example: Tsai, et al., 2010; Avila, et al., 2018). So, in general, when compared with previous empirical results, the MAPEs are moderately fine.

In order to reduce the uncertainty of the forecasts and improve accuracy, data that is more frequently recorded should be used instead of annual data. Either quarterly data that is seasonally adjusted or monthly data could yield estimations that are closer to actual values. Referring to the pioneering work of Lee et al. (2002), we would suggest using a couple hundreds of frequently recorded data points. If one were to do an annual forecast, there should be at least 50 observations in the sample. Having obtained a better sample, the dynamic method of forecasting should be employed. As the dynamic technique inputs predicted values to compute subsequent predictions, using it would help gain more insight into the performance of the Lotka – Volterra Competition model. If the confidence bands are not extensively enlarged over time but instead appear more restrained and stable, then it could be inferred that the model performs well as the predicted values are so close to the actual ones, the error terms are small enough to not make subsequent estimations deviate far from real observations.

If there is sufficient data points, another plausible improvement could be made by performing separate forecasts for periods prior to and after where the demand lines cross. Such analyses would give insight into whether there are fluctuations in the competitive states. It has been shown in the first research of that the parameters changed in different time spans, meaning that the interrelationship between 2 species changed as well. In detail, the paper found that the Korean stock markets KSE and KOSDAQ initially shared a predator-prey relationship, then symbiotic, and finally pure competition (Lee, et al., 2005). Additionally, in the case of Guatemala, mobile and fixed-line services changed from pure competition to amensalism (Avila, et al., 2018).To perform these separated analyses, it requires a large number of observations.

It should also be of interest for subsequent research to use the estimated parameters to employ an out-of-sample forecast to better gauge the performance of the Lotka-Volterra equations. Similar to the works of Lee, et al. (2005) and Kim et al. (2006), this could be done by firstly running the early portion of the data through the Gauss-Newton non-linear least squares method to yield the parameter estimations. Then, using the same parameters, produce forecasts for the next 1 to 5 data points. By comparing the forecasts with the actual observations, we could gauge whether the model could truly generate adequate predictions for unknown periods.

Although the parameter estimations in this study have R-squares over 98%, indicating that our independent variables can already explain a major proportion of the dependent variable, creating an additional regression would be beneficial for examining what factors other than competition could lead to increase in consumption of mobile phones. Many previous works have presented promising nominations for independent variables. For example, Brenner & Wulf 2013 proposed using GDPpc converted into international dollars using purchasing power parity and the ratio of households with fixed broadband access. Most of their R-squares were significantly above 95%, with an exception of 82.8% in one case (Brenner & Wulf, 2013). In addition to competitiveness and GDPpc, another study model penetration rate by inputing investment, which was then indicated to be a statistically significant parameter. The study also classified all possible independent variables into 3 groups: factors related to the economy (population, demography etc.), market (tariff, regulation etc.) , and technology (establishment of digital technology, number of fixed-lines etc.) (Avila, et al., 2018). Such indicators could make excellent candidates for a regression model. It should be noted that, if such tests are to be carried out, it should be mentioned that the R-square will certainly be lower if the regression is not a time-series model.

2. On the equilibrium analysis

Although the interactive behaviors that were inferred from the equilibrium analyses seem to be supported by real market trends, they could have been generated by a different method with the Jacobian matrix and the Lyapunov equation. Despite its complexity, this alternative has been featured in and endorsed by a great many of existing articles (See for example: Kreng & Wang, 2010; Kalogiratou, et al., 2013; Tsai, 2017; Wang & Wang, 2017). For a linaer system, z = Az so the Jacobian matrix at the equilibrium point should be:

$$\mathbf{A} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial X} & \frac{\partial f_1}{\partial Y} \\ \frac{\partial f_2}{\partial X} & \frac{\partial f_2}{\partial Y} \end{bmatrix}\right|_{(X,Y)=(X^*,Y^*)}$$
$$= \left.\begin{bmatrix} a_1 - 2b_1 X - c_1 Y & -c_1 X \\ -c_2 Y & a_2 - 2b_2 Y - c_2 X \end{bmatrix}\right|_{(X,Y)=(X^*,Y^*)}$$

Unlike our previous analysis where only the movements of the points are explained by the maximum capacity and the growth functions, the Jacobian eigenvalues give insight into how each variable change at the equilibrium point. Moreover, to see if the equilibrium point of this system is asymptotically stable, the Lyapunov function could be used. As this study is more concerned about whether the population theories from the LVC model could explain technological changes, and as this alternative is more mathematically demanding, the Jacobian matrix and the Lyapunov function were not called for. They are, nevertheless, better for demonstrating whether the markets will converge to the equilibrium, so they should be employed if possible.

F. Conclusion

This study used the Lotka-Volterra model to perform in-sample forecasts of the subscription for the mobile phone market and the telephone market, and, more importantly, determine their reciprocal influence. Empirical results show that the model performs well despite having a limited sample size. The theoretical conclusions on the relationship types between the markets in each economy were in accordance with the behaviors of historical data. In the future, the accuracy of the estimations could be improved by the means discussed in our limitations. Also, various approaches could be taken to gain a deeper understanding of the determinants of mobile and land-line diffusion as well as their eventual interactive outcome.

In terms of academic contribution, his research is an addition to the mass existing knowledge on the usage of population biology frameworks in technology development and on the developing area of fixed-to-wireless broadband substitution. Growing countries where less research interest has been received could utilize this simple model to understand the changes in its industries, including telecommunications and others that experience product or technology diffusion. With respect to firms, this model could also be used to gauge the degree of competitiveness in their respective business environment. For instance, they could perform the same analysis on the penetration rate of iOs versus Android technology to predict market demand and bring forth appropriate strategies for their line of product. It should be noted that other than this 2-species model, there are extensions that include many more species that would be more appropriate for different environments.

In the future, macroeconomics effects must be considered to extend the discussion on the determinants of these markets' behaviors and their policy implications. Also, if possible, the Lotka - Volterra model should be compared with its counterparts, such as the Bass, the logistic, and the Gompertz model to see which one can outperform.

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